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ON THE CHARACTERIZATION OF SIMPLE GROUPS $B_n(q)$ AND $C_n(q)$

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ABSTRACT. Let G be a finite group and $\pi(G)$ be the set of prime divisors of the order of G. For $t \in \pi(G)$ denote by $n_t(G)$ the order of a normalizer of t- Sylow subgroup of G and put $n(G) = \{n_t(G) \mid t \in \pi(G)\}$. In this talk, we discuss about an answer to the following problem for the simple groups of Lie type B_n, C_n:

Let L be a finite non-abelian simple group and G be a finite group with $n(L) = n(G)$. Is it true that $L \cong G$?

Also, we discuss about difference between orders of the solvable subgroups of the non-isomorphic simple groups $B_n(q)$ and $C_n(q)$.

1. INTRODUCTION

Characterization by orders of Sylow normalizers has first been considered by Bi in 1992 (see [2]). It is known that if G is $A_n(q)$, ${}^2A_n(q)$, $C_2(q)$, ${}^2D_n(q)$, alternating group, Mathieu simple groups, Janko groups and $Sz(2^{2m+1})$, then G is characterizable by orders of Sylow normalizers. In this talk, we discuss that if $n = 2$ or $q \not\equiv \pm 1 \pmod 8$, then $B_n(q)$ and $C_n(q)$ are characterizable by orders of Sylow normalizers and otherwise, $B_n(q)$ and $C_n(q)$ are $2-$recognizable by orders of Sylow normalizers.

For a finite group G, let $\mathrm{Ord}(\mathsf{S}_{sol}(G))$ be the set of orders of its solvable subgroups. The following conjecture was proposed by S. Abe and N. Iiyori [1]: Let G be a finite group and S be a non-abelian simple group. Then $G \cong S$ if and only if $\mathrm{Ord}(\mathsf{S}_{sol}(G)) = \mathrm{Ord}(\mathsf{S}_{sol}(S))$.

It was proved that if S is a simple group and G is a finite group such that $\mathrm{Ord}(\mathsf{S}_{sol}(G)) = \mathrm{Ord}(\mathsf{S}_{sol}(S))$, then $G \cong S$ or $\{G, S\} = \{B_n(q), C_n(q)\}$, where $n \geq 3$ and q is an odd prime power (see [3]). The purpose of this talk is to prove that the $\mathrm{Ord}(\mathsf{S}_{sol}(B_n(q)))$ and $\mathrm{Ord}(\mathsf{S}_{sol}(C_n(q)))$ are distinct.

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2. Main results

Theorem 2.1. *Let $S_{n,q} \in \{B_n(q), C_n(q)\}$ and let G be any finite group such that $n(S_{n,q}) = n(G)$. If $q \not\equiv \pm 1 (mod\ 8)$, then G is isomorphic to $S_{n,q}$. Otherwise, G is isomorphic to $B_n(q)$ or $C_n(q)$.*

Theorem 2.2. *Let q be an odd prime power and $n \geq 3$. If $S \in \{B_n(q), C_n(q)\}$, then there are infinite pairs $\{(n,q)\}$ such that*

$$\mathrm{Ord}(\mathcal{S}_{sol}(B_n(q))) \neq \mathrm{Ord}(\mathcal{S}_{sol}(C_n(q))).$$

Theorem 2.3. *Let q be an odd prime power and $n \geq 3$. For the infinite pairs $\{(n,q)\}$, simple groups $B_n(q)$ and $C_n(q)$ are characterizable by orders of solvable subgroups.*

References

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