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SEMISYMMETRIC CUBIC GRAPHS OF ORDER $4p^3$

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Abstract. An undirected graph without isolated vertices said to be semisymmetric if its full automorphism group acts transitively on its edge set but not on its vertex set. In this paper we prove that for every prime p, there is no semisymmetric cubic graph of order $4p^3$.

1. Introduction

Throughout this paper, graphs are assumed to be finite, simple, undirected and connected. For a graph X, we denote by $V(X)$, $E(X), A(X)$ and $\mathrm{Aut}(X)$ its vertex set, edge set, arc set and automorphism group, respectively. For u, $v \in V(X)$, denote by uv the edge incident to u and v in X, and by $N_X(u)$ the *neighborhood* of u in X, that is, the set of vertices adjacent to u in X. A graph is *vertex-transitive, edge-transitive* and *arc-transitive* if its automorphism group acts transitively on the vertices, edges and arcs, respectively. An arc-transitive graph is called *symmetric*.

Let N be a subgroup of $Aut(X)$. The *quotient graph* X/N or X_N of X relative to N is defined as the graph such that the set Σ of N-orbits in $V(X)$ is the vertex set of X/N and $B, C \in \Sigma$ are adjacent if and only if there exist $u \in B$ and $v \in C$ such that $uv \in E(X)$.

A graph $\widetilde{X}$ is called a *covering* of a graph X with projection $p : \widetilde{X} \to X$, if p is a surjection from $V(\widetilde{X})$ to $V(X)$ such that $p\mid_{N_{\widetilde{X}}(\tilde{v})} : N_{\widetilde{X}}(\tilde{v}) \to N_X(v)$ is a bijection for any vertex $v \in V(X)$ and $\tilde{v} \in p^{-1}(v)$. The *fibre* of an edge or a vertex is its preimage under p. If $\widetilde{X}$ is connected, then any two vertex or edge fibres are of the same cardinality n. This number is called the *fold number* of the covering, and we say that p is an n-fold covering. A covering $\widetilde{X}$ of X with a projection p is said to be *regular* (or *K-covering*) if there is a semiregular subgroup K of the automorphism group $\mathrm{Aut}(\widetilde{X})$ such that graph X is isomorphic to the quotient graph $\widetilde{X}/K$, say by h, and the quotient map $\widetilde{X} \to \widetilde{X}/K$ is the composition ph of p and h.

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Covering techniques have long been known as a powerful tool in topology and graph theory. The study of semisymmetric graphs was initiated by Folkman [5]. Semisymmetric graphs of order $2pq$ and semisymmetric cubic graphs of orders $2p^3$ and $6p^2$ are classified in [4, 7, 6], and also in [1] it is proved that every edge-transitive cubic graph of order $8p^2$ is vertex-transitive. In [3], it is given an overview of known families of semisymmetric cubic graphs.

The following proposition is a special case of [6, Lemma 3.2].

Proposition 1.1. Let X be a connected semisymmetric cubic graph with bipartition sets $U(X)$ and $W(X)$. Moreover, suppose that N is a normal subgroup of $A :=\text{Aut}(X)$. If N is intransitive on bipartition sets, then N acts semiregularly on both $U(X)$ and $W(X)$, and X is an N-regular covering of a A/N-semisymmetric graph.

We quote the following propositions.

Proposition 1.2. [7, Proposition 2.4] The vertex stabilizers of a connected G-edge-transitive cubic graph X have order $2^r \cdot 3$, $r \geq 0$. Moreover, if u and v are two adjacent vertices, then $|G : \langle G_u, G_v\rangle| \leq 2$, and the edge stabilizer $G_u \cap G_v$ is a common Sylow 2-subgroup of G_u and G_v.

Proposition 1.3. [8] Every both edge-transitive and vertex-transitive cubic graph is symmetric.

Proposition 1.4. [2] If $\tilde{X}$ is a bipartite covering of a non-bipartite graph X; then the fold number is even.

2. Main results

Lemma 2.1. *Suppose that X is a semisymmetric cubic graph of order $4p^3$, where $p \geq 7$ is a prime. Set $A :=Aut(X)$, moreover suppose that $Q := O_p(A)$ be the maximal normal p-subgroup of A. Then $|Q| = p^3$.*

Theorem 2.2. *Let p be a prime. Then there is no semisymmetric cubic graph of order $4p^3$.*

References

[1] M. Alaeiyan and M. Ghasemi, Cubic edge-transitive graphs of oredr $8p^2$, *Bull. Austral. Math. Soc.*, **77** (2008), 315-323.

[2] D. Archdeacon, J.H. Kwak, J. Leec and M.Y. Sohn, Bipartite covering graphs, *Discrete Mathematics*, **214** (2000), 51-63.

[3] M. Conder, A. Malnic, D. Marusic and P. Potocnik, A census of semisymmetric cubic graphs on up to 768 vertices, *J. Algebr. Comb.*, **23** (2006), 255-294.

[4] S. F. Du and M. Y. Xu, A classification of semisymmetric graphs of order $2pq$, *Com. in Algebra*, **28(6)** (2000), 2685-2715.

[5] J. Folkman, Regular line-symmetric graphs, *J. Combin. Theory*, **3** (1967), 215-232.

[6] Z. Lu, C. Q. Wang and M. Y. Xu, On semisymmetric cubic graphs of order $6p^2$, *Science in Chaina Ser. A Mathematics*, **47** (2004), 11-17.

[7] A. Malnic, D. Marusic and C. Q. Wang, Cubic edge-transitive graphs of order $2p^3$, *Discrete Math.*, **274** (2004), 187-198.

[8] W. T. Tutte, *Connectivity in graphs*, Toronto University Press, 1966.