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On Graphs Whose Second Largest eigenvalue equals 1 [1]

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Abstract

Let G be a graph of order n and let μ be an eigenvalue of multiplicity m. A star complement for μ in G is an induced subgraph of G of order $n-m$ with no eigenvalue μ. Here we first identify among Cactus graphs, complete three partite graphs $K_{n,n,n}$, and bicyclic graphs which can be star complement for 1 as the second largest eigenvalue. Using the graphs obtained, we next search for their maximal extensions, either by theoretical means, or by computer aided search.

1 Introduction

Let G be a finite simple graph with an eigenvalue μ of multiplicity m; in other words, a $(0,1)$-adjacency matrix G has a μ-eigenspace of dimension m. An m-subset X of $V(G)$ is called a star set for μ in G if μ is not an eigenvalue of $G-X$. The induced subgraph $H = G-X$ is said to be a star complement for μ in G. If G has star complement H for μ, and G is not a proper induced subgraph of some other graph with star complement H for μ, then G is a maximal graph with star complement H for μ.

A Coctus graph is a connected graph in which any two simple cycles have at most one vertex in common; Equivalently, every edge in such a graph may

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belong to at most one cycle. In this paper we will determine all cactus graphs which can be star complement for $\lambda_2 = 1$ (second largest eigenvalue equals 1). Connected graphs in which the number of edges equals the number of vertices plus one are called bicyclic graph. The graphs with unicyclic graphs as a star complement were discussed in [2]. Here we show all bicyclic graphs which can be star complements for $\lambda_2 = 1$. Let $K_{n,n,n}(a, b, c)$ be a graph obtained from $K_{n,n,n}$ by introducing a new vertex and joining it to a vertices of first part, b vertices of second part and c vertices of third part. Let $A = a + b + c$ and $B = ab+ac+bc$, here we discuss graphs with complete threepartite graphs $K_{n,n,n}$ as a star complement for 1 as the second largest eigenvalue where $A \geq B$.

2 Main results

Theorem 2.1 *All Cactus graphs which are star complement for $\lambda_2 = 1$ are trees and unicyclic graphs.*

Theorem 2.2 *A bicyclic graph H is a star complement for $\lambda_2 = 1$if and only if it is one of the graphs depicted in Fig.1.*

Theorem 2.3 *The strongly regular graphs don't have strongly regular graphs as a star complement for $\lambda_2 = 1$.*

Theorem 2.4 *If $H = K_{n,n,n}$ is a star complement for $\lambda_2 = 1$ and A and B be as defined above then $n = 2, 3, 5$ if and only if $A \geq B$.*

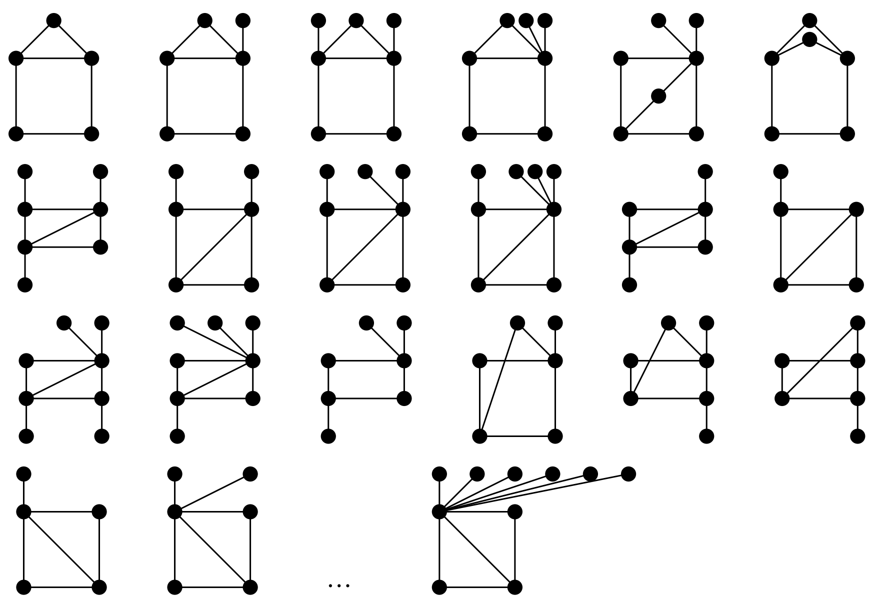

Fig.1

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