Tarbiat Moallem University, 20^{th} Seminar on Algebra,
2-3 Ordibehesht, 1388 (Apr. 22-23, 2009) pp 222-224

SOME RESULTS ON PRIME AND MAXIMAL SUBNEXUSES OF A NEXUS

L. TORKZADEH

Department of Mathematics
Islamic Azad university, Kerman Branch,
Kerman, Iran abhasan@mail.uk.ac.ir
(Joint work with A. Hasankhani)
Dept. of Math. , Islamic Azad university, Kerman Branch, Kerman, Iran

ABSTRACT. In this paper the notions of prime and maximal subnexuses of a nexus are defined and the relationship between of them are investigated. In particular, all prime and maximal subnexuses of a nexus are characterized.

1. INTRODUCTION AND PRELIMINARIES

The space structure research center of university of Surrey was founded by Z.S. Makoswski as a part of civil engineering in 1963. The aim of the Center is to carry out research into the design and analysis of space structures. Space structures include structural forms such as single and double layer girds, barrel vaults, shells and various forms of tension structures.
The basic idea of a nexus has been further developed as a mathematical object for general use. The aim of recent study has been to evolve a mathematical object that allows complex processes on groups of mathematical objects to be formulated with ease of elegance. This notion is very useful for study of space structure. In this paper, we define the notions of prime and maximal subnexuses of a nexus and investigate some results as mentioned in abstract.

Note that, in this paper $\aleph \cup \{0\}$ is denoted by $\aleph^*$, where $\aleph$ is the set of natural numbers.

Definition 1.1. (i) An address is a sequence of $\aleph^*$ such that $a_k = 0$ implies that $a_i = 0$, for all $i \geq k$. The sequence of zero is called the empty address and denoted by $()$. In other word, every nonempty address is of the form $(a_1, a_2, ..., a_n, 0, 0, ...)$ where a_i and n are belong to $\aleph$. Hereafter this address will be denoted by $(a_1, a_2, ..., a_n)$.
(ii) A nexus N is a set of address with the following properties:
(1) if $(a_1, a_2, ..., a_{n-1}, a_n) \in N$, then $(a_1, a_2, ..., a_{n-1}, t) \in N$, for all $0 \leq t \leq a_n$,
(2) if $(a_1, a_2, ...) \in N$, then $(a_1, a_2, ..., a_n) \in N$, for all $n \in N$.

Definition 1.2. *Let N be a nexus. A subset S of N is called a subnexus of N provided that S itself is a nexus.*

Definition 1.3. Let $w \in N$. The level of ω is said to be :
(i) n, if $w = (a_1, a_2, ..., a_n)$, for some $a_n \in N$,
(ii) ∞, if w is an infinite sequence of N,

(iii) 0, if $w = ()$.
The level of w is denoted by $l(w)$.

Definition 1.4. Let $w = (a_1, a_2, ...)$ and $v = (b_1, b_2, ...) \in N$. Then $w \leq v$, if $l(w) = 0$ or one of the following cases satisfies:
Case 1: If $l(w) = 1$, that is, $w = (a_1)$, for some $a_1 \in \aleph$ and $a_1 \leq b_1$.
Case 2: If $1 < l(w) < \infty$, then $l(w) \leq l(v)$ and $a_{l(w)} \leq b_{l(w)}$ and $a_i = b_i$ for all $i < l(w)$.
Case 3: If $l(w) = \infty$, then $w = v$.

Theorem 1.5. *Let N be a nexus. Then $(N, \wedge)$ is a semilattice, where for each $w, v \in N$*
(i) *if $v = ()$ or $w = ()$, then $v \wedge w = ()$,*
(ii) *if $v \neq ()$, $w \neq ()$ and $n \in N$ is the least integer such that $a_n \neq b_n$, then $v \wedge w = (a_1, a_2, ..., a_n \wedge b_n)$.*

2. Prime subnexuses of a nexus

Definition 2.1. A proper subnexus P of a nexus N is said to be a prime subnexus of N if $a \wedge b \in P$ implies that $a \in P$ or $b \in P$, for any $a, b \in N$.

For example, consider the nexus $N = \{(), (1), (2), (1,1), (1,2), (1,3), (2,1), (2,2)\}$ The subnexus $P = \{(), (1), (2), (2,1), (2,2)\}$ is prime subnexus of N, but the subnexus $K = \{(), (1), (2), (1,1), (2,1)\}$ is not prime, because $(1,2) \wedge (2,2) = (1) \in K$ but neither $(1,2)$ nor $(2,2)$ belong to K.

Lemma 2.2. *The trivial subnexus of N, that is, $\{()\}$ is prime.*

Let T and S be two subsets of a nexus N. We define the set $T \wedge S = \{t \wedge s | \ t \in T$ and $s \in S\}$.

Theorem 2.3. *Let P be a proper subnexus of N. Then the following are equivalent:*
(i) *P is a prime subnexus of N,*
(ii) *$K_1 \wedge K_2 \subseteq P$ implies $K_1 \subseteq P$ or $K_2 \subseteq P$, for any subnexuses K_1 and K_2 of N,*
(iii) *$< a > \wedge < b > \subseteq P$ implies $a \in P$ or $b \in P$, for any $a, b \in N$,*
(iv) *$K_1 \wedge K_2 = P$ implies $K_1 = P$ or $K_2 = P$, for any subnexuses K_1 and K_2 of N.*

Definition 2.4. Let $a = (a_1, a_2, ..., a_k)$ be an address of N.
(i) The set $\{(a_1, a_2, , a_k, a_{k+1}, ..., a_n) \in N | \ a_{k+i} \in \aleph$ for $i = 1, 2, ..., n-k\}$ is called the 'remus' of a and is denoted by r_a. The 'remus' of a is not including a.
(ii) the set $\{b \in N | \ a \leq b\}$ is called the 'super tect' of a and is denoted by R_a. The 'super tect' of a is including of a.
(iii) Let S be a non-empty subset of N, then $r_S = \bigcup_{a \in S} r_a$ and $R_S = \bigcup_{a \in S} R_a$.

Theorem 2.5. *Let P be a subnexus of N. Then P is a prime subnexus if and only if $N - P$ is closed under meet operation.*

Theorem 2.6. *Let S be a nonempty subset of N. Then $N - R_S$ and $N - r_S$ are prime subnexuses of N.*

Theorem 2.7. *Every prime subnexus of N is the form of $N - R_a$, for some $a \in N$.*

Theorem 2.8. *Suppose that, a and b are not comparable addresses of N. Then,*
(i) *$R_a \cap R_b = \emptyset$,*
(ii) *$R_a \wedge R_b = \{a \wedge b\}$.*

Theorem 2.9. *Let a and b be two comparable addresses of N such that $a < b$. If a and b are the same level, then $r_a \wedge r_b = \{a\}$.*

Theorem 2.10. *Let a and b be two addresses of N and let $P_1 = N - R_a$ and $P_2 = N - R_b$ be two prime subnexuses of N. Then,*
(i) *if a and b are two comparable addresses, then $P_1 \cap P_2$ is a prime subnexus of N,*
(ii) *if a and b are not comparable addresses, then $P_1 \cap P_2$ is not a prime subnexus of N and $P_1 \cup P_2 = N$.*

Theorem 2.11. *Nexus N is linearly ordered if and only if every proper subnexus of N is prime.*

3. Maximal subnexuses of a nexus

Definition 3.1. A maximal subnexus of N is a subnexus U, not equal to N, such that there are no subnexus in between U and N. Consider

$$N = \{(),(1),(2),(1,1),(1,2),(2,1),(2,2),(2,3)\}$$

and a subnexus $U = \{(),(1),(2),(1,1),(1,2),(2,1),(2,2)\}$. Then it is easy to check that U is maximal subnexus of N. But the subnexus $T = \{(),(1),(2),(2,1),(2,2),(2,3)\}$ is not maximal subnexus, because there exist a subnexus

$$K = \{(),(1),(2),(1,1),(2,1),(2,2),(2,3)\}$$

such that, $T \subset K$ and $K \subset N$.

Theorem 3.2. *Let N do not have any maximal address . Then N do not have any maximal subnexus.*

Theorem 3.3. *Let $m = \{m_i | \ i \in I\}$ be set of all maximal addresses of N. Then every maximal subnexus of N is the form of $M = N - \{m_i\}$, for some $m_i \in m$. Furthermore, the number of maximal subnexus of N is equal to the number of maximal addresses of N, that is, $|m|$.*

Theorem 3.4. *Every maximal subnexus of N is prime.*

Theorem 3.5. *Every prime subnexus of N is maximal if and only if $N = \{(),(1)\}$.*

Theorem 3.6. *N is cyclic if and only if N has just one maximal subnexus.*

References

[1] M. Bolurian, A. Hasankhani and H. Nooshin, *Nexus algebra*, Submitted.

[2] H. Nooshin and P. Disney, *Formex configuration processin I*, International journal of space structures,15 (2000), no. 1, 1-52.

[3] H. Nooshin and P. Disney, *Formex configuration processin II*, International journal of space structures,16 (2001), no. 1, 1-56.

[4] H. Nooshin and P. Disney, *Formex configuration processin III*, International journal of space structures,17 (2002), no. 1, 1-50.