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A NOTE ON SUBGROUPS OF DIVISION RINGS

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Abstract. Let D be a division ring. An easy consequence of a famous result of Herestein asserts that if D^* (the multiplicative group of D) is an FC group (group with finite conjugacy classes), then it is abelian. A similar result is also true for any FC subnormal subgroup N of D^* (see lemma 1 of below). Now, let M be a maximal subgroup of N. In this talk, I prove that if M is an FC group, then it is abelian.

1. Introduction

Let D be a division ring. Denote by D^* the multiplicative group of D. By a result of Herestein, it is known that any non-central element of D^* has infinitely many conjugates in D^*. Therefore if every element of D^* has only a finite number of conjugates (such a group is called an FC group), then it is abelian. Due to the Cartan-Brauer-Hua Theorem (and some generalization of it), it is known that the structure of any subnormal subgroup of D^* is similar to the structure of D^*. Specially, we prove that if a subnormal subgroup of D^* is an FC group, than it is abelian (lemma 2.1). Now, as with the subnormal subgroup N of D^*, one would like to know the structure of maximal subgroups of N and how they are similar to D^*. In this talk, I prove that if a maximal subgroup M of a subnormal subgroup of D^* is an FC group, then M is also abelian.

2. Main results

Lemma 2.1. *Let D be a division ring with center F, and N a subnormal subgroup of D^*. If N is an FC group, then $N \subseteq F^*$.*

The following lemma is a generalization of Lemma 9 of [1].

Lemma 2.2. *Let D be an infinite division ring with center F, N a non-central subnormal subgroup of D^*, and M a maximal subgroup of N. If $|M/M \cap F^*| < \infty$, then M is abelian.*

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We shall say that a group G is abelian-by-finite (center-by-finite) if there is an abelian normal (central) subgroup A of G such that G/A is a finite group.

Theorem A [5, p. 76]. A linear FC group is center-by-finite.

Lemma 2.3. *Let D be an infinite division algebra, finite dimensional over its center F. Suppose that N be a subnormal subgroup of D^* and M a maximal subgroup of N. If M is an FC group, then M is abelian.*

Let R be an F-algebra over a field F. We say that R satisfies in a polynomial identity (or R is a PI-ring), if there exists a nonzero polynomial $p(x_1, \ldots, x_m) \in F[x_1, \ldots, x_m]$ such that for any $a_1, \ldots, a_m \in R$ we have $p(a_1, \ldots, a_m) = 0$.

Theorem B [2, Theorem 2]. Let D be a division ring with center F, N a non-central subnormal subgroup of D^* and M a non-abelian maximal subgroup of N such that $F(M)$ satisfies a polynomial identity. If $C_D(M)\backslash F$ contains an algebraic element over F or $C_D(M) = F$, then $[D : F] < \infty$.

Theorem 2.4. *Let D be a division ring and M be a maximal subgroup of N which is a subnormal subgroup of D^*. If M is an FC group, then M is abelian.*

Corollary 2.5. *Let D be a division ring and M be a maximal subgroup of N which is a subnormal subgroup of D^*. If M is non-abelian, then $M/Z(M)$ is an infinite group.*

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