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A PRESENTATION FOR EXTENDED AFFINE LIE ALGEBRAS

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(Joint work with S. Azam and H. Yamane)

1. Introduction

For a complex finite dimensional simple Lie algebra $\mathscr{G}$ and a field $\mathbb{K}$, one can define a Lie algebra $\mathscr{G}(\mathbb{K}) := \mathscr{G}_{\mathbb{Z}} \otimes_{\mathbb{Z}} \mathbb{K}$ over $\mathbb{K}$ where $\mathscr{G}_{\mathbb{Z}}$ is the *Chevalley $\mathbb{Z}$-form* of $\mathscr{G}$ with respect to a given *Chevalley basis* of $\mathscr{G}$. In the case that the rank of $\mathscr{G}$ is greater than 1 and $\mathrm{ch}(\mathbb{K}) \neq 2,3$, Stienberg [St] proves that $\mathscr{G}(\mathbb{K})$ is centrally closed and gives a presentation of $\mathscr{G}(\mathbb{K})$ by generators and relations. Kassel [K] generalizes this concept by considering a unital commutative algebra A over a commutative ring R in place of the field $\mathbb{K}$ and defines the Lie algebra $\mathscr{G}(A) := \mathscr{G}_{\mathbb{Z}} \otimes_{\mathbb{Z}} A$ over R. He proves that the universal covering algebra of $\mathscr{G}(A)$ is $\tilde{\mathscr{G}}(A) := \mathscr{G}(A) \oplus C$ where C is linearly isomorphic to Ω^1_A/dA, the module of *Kähler differentials* of A modulo *exact forms*. He also gives a presentation of $\tilde{\mathscr{G}}(A)$ by generators and relations. When $R = \mathbb{C}$ and A is the algebra of Laurent polynomials in $n-$variables, the algebra $\tilde{\mathscr{G}}(A)$ is called, by Moody, Rao and Yokonuma [MRY], an $n-$*toroidal* Lie algebra. They give an abstract infinite presentation of a $2-$toroidal Lie algebra in terms of generators and relations involving the extended Cartan matrix of $\mathscr{G}$. They use their presentation to construct a great number of representations of $\tilde{\mathscr{G}}(\mathbb{C}[t_1^{\pm 1}, t_2^{\pm 1}])$ for a simply laced algebra $\mathscr{G}$. Saito and Yoshii [SaY] introduce a class of Lie algebras whose cores are $2-$toroidal Lie algebras. They call their class *elliptic* Lie algebras as they are used in the study of *elliptic singularities*. They give a Serre-type presentation of a simply laced elliptic Lie algebra in term of the elliptic *Dynkin diagram* (R,G) attached to its *elliptic root system* R (an extended affine root system of nullity 2) with *marking* G which is a rank 1 subspace of the radical of the semi-positive symmetric bilinear form defining R. Yamane [Ya] extends the presentation given by Saito and Yoshii to elliptic Lie algebras in general. More precisely, he gives a Serre-type theorem for the elliptic Lie algebras associated to the (reduced marked) elliptic root systems with rank greater than 2. A toroidal Lie algebra is centrally isogenous to the centerless core of an *extended affine Lie algebra* which is in turn a Lie torus. Now the question is whether one could find a (finite) presentation of the universal covering algebra of a Lie torus for a given nullity and type. In this work we give an affirmative answer to this question.

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2. Main results

Theorem 2.1. *The universal covering algebra of a Lie torus of type $X \neq A_1, C$, is a finitely presented Lie algebra.*

References

[K] C. Kassel, *Kähler differentials and coverings of complex simple Lie algebras extended over a commutative algebra,* J. Pure Appl. Algebra **34** (1984), no. 2-3, 265–275.

[MRY] R. V. Moody, S. E. Rao and T. Yokonuma, *Toroidal Lie algebras and vertex representations.* Geom. Dedicata **35** (1990), no. 1-3, 283–307.

[SaY] K. Saito and D. Yoshii, *Extended affine root systems IV (Simply-laced Elliptic Lie Algebras)*, Publ. Res. Inst. Math. Sci. **36** (2000), no. 3, 385–421.

[St] R. Steinberg, *Générateurs, relations et revêtements de groupes algébriques,* Colloq. Théorie des Groupes Algbriques (1962) 113–127.

[Ya] H. Yamane, *A Serre-type theorem for the Elliptic Lie algebras with rank* ≥ 2, Publ. Res. Inst. Math. Sci. **40** (2004), no. 2, 441–469.