

CENTRAL AUTOMORPHISMS OF SEMIDIRECT PRODUCT OF FINITE GROUPS

HAMID MOUSAVI AND AMIR SHOMALI

Department of Mathematics
University of Tabriz
Tabriz, Iran
hmousavi@tabrizu.ac.ir
a.shomali@tabrizu.ac.ir

ABSTRACT. In this paper we find the order and structure of $\text{Aut}_c(G)$ for $G = K \rtimes H$ by imposing some conditions on H and K . As a result, we show that if M is a non-normal maximal subgroup of a solvable group G , such that all central automorphisms of G fix $\text{Core}_G(M)$ pointwise, then $\text{Aut}_c(G)$, is meta-abelian.

1. INTRODUCTION

An automorphism σ of a group G is central if σ commutes with every automorphism in $\text{Inn}(G)$, the group of inner automorphisms of G , or equivalently, if $g^{-1}\sigma(g)$ lies in the center $Z(G)$ of G , for all g in G . The central automorphisms fix the commutator subgroup G' of G pointwise, and form a normal subgroup, denoted by $\text{Aut}_c(G)$, of the full automorphism group $\text{Aut}(G)$. The group of central automorphisms of a finite group G is of great importance in the investigation of $\text{Aut}(G)$, and has been studied by several authors (see, for example, [1-6]).

In [1] Adney and Yen has shown that if G is a finite purely non-abelian group then $|\text{Aut}_c(G)| = |\text{Hom}(G/G', Z(G))|$. Suppose $G = K \times H$ where K is purely non-abelian and H an abelian subgroup of G . In [5] Jamali and Jafari introduced some subgroups for $\text{Aut}_c(G)$ to find order and structure of $\text{Aut}_c(G)$.

In this paper we find the order and structure of $\text{Aut}_c(G)$ for $G = K \rtimes H$ by imposing some conditions on H and K . As a result, we show that if M is a non-normal maximal subgroup of a solvable group G , such that all central automorphisms of G fix $\text{Core}_G(M)$ pointwise, then $\text{Aut}_c(G)$, is meta-abelian.

In this talk, G is a finite group, π_H and π_K are the projection maps from G into H and K respectively and σ_H, σ_K are restriction of σ on H and K

2000 Mathematics Subject Classification: 20D15, 20D45.

keywords and phrases: Central Automorphism, Solvable group, Semidirect group.

respectively. Also we set:

$$\begin{aligned} R &= \{\pi_H \sigma_H \mid \sigma \in \text{Aut}_c(G)\} & S &= \{\pi_K \sigma_K \mid \sigma \in \text{Aut}_c(G)\} \\ T &= \{\pi_K \sigma_H \mid \sigma \in \text{Aut}_c(G)\} & U &= \{\pi_H \sigma_K \mid \sigma \in \text{Aut}_c(G)\} \end{aligned}$$

2. MAIN RESULTS

Theorem 2.1. *Let $G = K \rtimes H$ be a semidirect product, such that $(|H|, |K|) = 1$. then $\text{Aut}_c(G) \cong R \times S$. Furthermore, if $Z(H)$ acts trivially on K then $\text{Aut}_c(G) \cong \text{Aut}_c(H) \times S$. In particular if $G = K \times H$, then $\text{Aut}_c(G) \cong \text{Aut}_c(H) \times \text{Aut}_c(K)$.*

Corollary 2.2. *Let G be a solvable group. If M is a non-normal maximal subgroup of index p^t such that $p \nmid |M/C|$ and $(|M/C|, |C|) = 1$, where $C = \text{Core}_G(M)$ then we can write $G = K \rtimes H$, for which $(|K|, |H|) = 1$ and $H \leq M$. Hence $\text{Aut}_c(G) \cong R \times S$.*

Theorem 2.3. *Let $\frac{G}{N} = \frac{K}{N} \rtimes \frac{H}{N}$ be a semidirect product and the central automorphisms fix the subgroup N of G pointwise and $(|\frac{H}{N}|, |\frac{K}{N}|) = 1$. then $\text{Aut}_c(G) \cong R \times S$.*

Theorem 2.4. *Let $G = K \rtimes H$ be a semidirect product, such that $\mathcal{C}_K(H) = 1$. Then $\text{Aut}_c(G) \cong R \times U$.*

Theorem 2.5. *Let $\frac{G}{N} = \frac{K}{N} \rtimes \frac{H}{N}$ be a semidirect product and the central automorphisms fix the subgroup N of G pointwise and $\mathcal{C}_{K/N}(H/N) = 1$. then $\text{Aut}_c(G) \cong R \times U'$ where $U' = \{f_\sigma \mid f_\sigma \in \text{Hom}(K, Z(G) \cap H), \sigma \in \text{Aut}_c(G)\}$.*

Corollary 2.6. *Let G be a solvable group with non-normal maximal subgroup M . If all central automorphisms fix $\text{Core}_G(M)$ pointwise, then $\text{Aut}_c(G) \cong R$ and so is abelian.*

Theorem 2.7. *Let $G = K \rtimes H$ be a semidirect product, where K is purely non-abelian and $Z(H)$ acts trivially on K . then $\text{Aut}_c(G) \cong RTUS$.*

REFERENCES

- [1] J. E. Adney and T. Yen, Automorphisms of a p -groups, *Illinois J. math.* **9** (1965), 137-143.
- [2] M. J. Curran and D. J. McCaughan, Central automorphisms that are almost inner, *Comm. Algebra*, **29(5)** (2001), 2081-2087.
- [3] B. Huppert, *Endliche Gruppen*, Vol. 1, Springer-Verlag, (1967).
- [4] M. H. Jafari, Elementary abelian p -group as central automorphism groups, *Comm. Algebra*, **34** (2006), 601-607.
- [5] M. H. Jafari and A. R. Jamali, On the occurrence of some finite groups in the central automorphism group of finite groups, *Math. Proc. Royal Irish Acad.*, **106A(2)** (2006), 139-148.
- [6] A. R. Jamali and H. Mousavi, On the central automorphism groups of finite p -groups, *Algebra Colloq.*, **9(1)** (2002), 7-14.